College Algebra

Chapter 2

Note:

This review is composed of questions similar to those in the chapter review at the end of chapter 2. This review is meant to highlight basic concepts from chapter 2. It does not cover all concepts presented by your instructor. Refer back to your notes, unit objectives, handouts, etc. to further prepare for your exam.

This review is available in alternate formats upon request.

Determine if the functions are linear or nonlinear. If the function is linear, determine the equation that defines y=f(x) or g(x)

x	-2	-0	1	3	5
y=f(x)	-4	2	5	11	17

x	-2	0	1	3	5
y=g(x)	-8	0	1	3	8

To determine if the function is linear, find the slope between each set of points. If all the slopes are the same, the function in linear. Remember slope $= \frac{y_2 - y_1}{x_2 - x_1}$ and linear functions are in the form y=mx+b

x	-2	-0	1	3	5
y=f(x)	-4	2	5	11	17

$= \frac{2-(-4)}{0-(-2)}$ $= 3$ $= 3$ $= 3$

$= \frac{6}{2} = 3$

The slope is the same between each point, so the f(x) is linear with slope, m, =3.
The y-intercept, b, is where x=0. In this case b=2.
The linear function is
f(x)=3x+2

x	-2	0	1	3	5
y=g(x)	-8	0	1	3	8

$= \frac{-2-0}{-8-0}$ $= 1$ $= 1$ $= \frac{5}{2}$

$= \frac{1}{4} =$

The slope is the not the same between each point, so g(x) is nonlinear.

Find the real zeros of each quadratic equation. What are the x-intercepts of the graph of the function.

$$f(x) = x^2 + x - 56$$
$$g(x) = 2x^2 - 6x - 5$$
$$h(x) = x^2 - 2x - 16$$

Zero's means x-intercepts when we set y=0 and solve for x. There are different ways to solve these functions. We will demonstrate different approaches .

Solving f(x) by factoring	Solve g(x) by using the quadratic formula	Solve h(x) by completing the square
$0 = x^2 + x - 56$ Multiples(factors) of -56: -1,56; 1,-56; -2,28; 2,-28; -4,14; 4,-14; -7,8; 7,-8 Only -7 and 8 add to 1 (-7+8)=1 $0 = (x-7)(x+8)$ Set each factor equal to zero and solve for x. x-7=0 x+8=0 x=7 x=-8 The x-intercepts are 7 and -8	$0 = 2x^2 - 6x - 5$ $A = 2$, b=-6, c=-5 $x = \frac{-(-6) \pm \sqrt{(-6)^2 - 4(2)(-5)}}{2(2)}$ $x = \frac{6 \pm \sqrt{36-(-40)}}{4}$ $x = \frac{6 \pm \sqrt{76}}{4}$ (simplify the root) $x = \frac{6 \pm \sqrt{4}\sqrt{19}}{4}$ $x = \frac{6 \pm 2\sqrt{19}}{4}$ $x = \frac{3 \pm \sqrt{19}}{2}$ The x-intercepts are $\frac{3 \pm \sqrt{19}}{2}$	$0 = x^2 - 2x - 16$ $16 = x^2 - 2x$ $16 + \_\_\_ = x^2 - 2x + \_\_\_$ Take half of the middle term (-2), square it and add to *both* sides. $16 + 1 = x^2 - 2x + 1$ $\left(\frac{-2}{2}\right)^2 = 1$ $17 = x^2 - 2x + 1$ (now factor) $17 = (x-1)(x-1)$ $17 = (x-1)^2$ (take square root) $\pm\sqrt{17} = x - 1$ $1 \pm \sqrt{17} = x$ The x-intercepts are $1 \pm \sqrt{17}$

Find f(x) = g(x)

$$f(x) = (x+7)^2 \quad g(x) = 9$$

$$f(x) = g(x)$$

Solve using the square root property

$$(x+7)^2 = 9$$

Take the square root of both sides

$$x + 7 = \pm 3$$

Subtract 7 from both sides

$$x = -7 \pm 3$$

$$x = -7 + 3 = -4$$
$$x = -7 - 3 = -10$$

Graph the function using transformations (shifting, stretching, and reflection).

$$f(x) = 2(x+1)^2 + 4$$

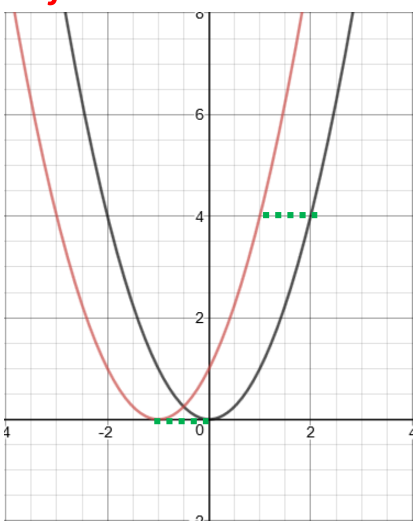
$$f(x) = 2(x+1)^2 + 4$$

stretch vertically by factor of 2

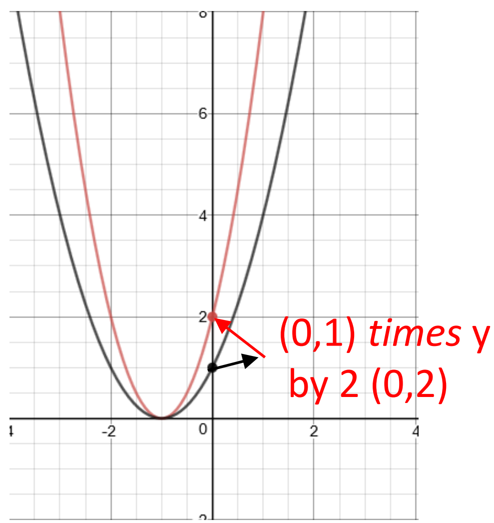
left one

up 4

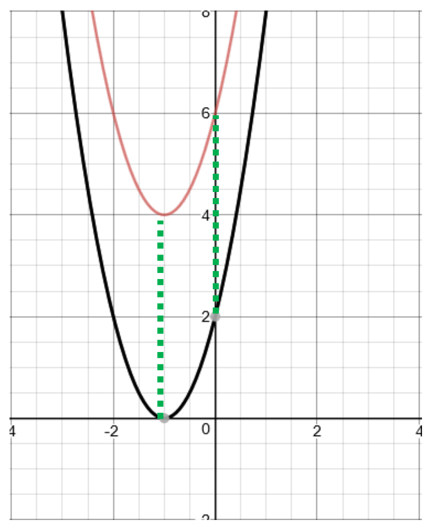
1. Horizontal shifting left one



2. Vertical Stretch by Factor of 2



3. Vertical shift up 4



Order for Transformations: 1. Horizontal shifts 2. Stretch/Shrink 3. Reflecting 4. Vertical Shifts

$$f(x) = x^2 - 2x - 8$$

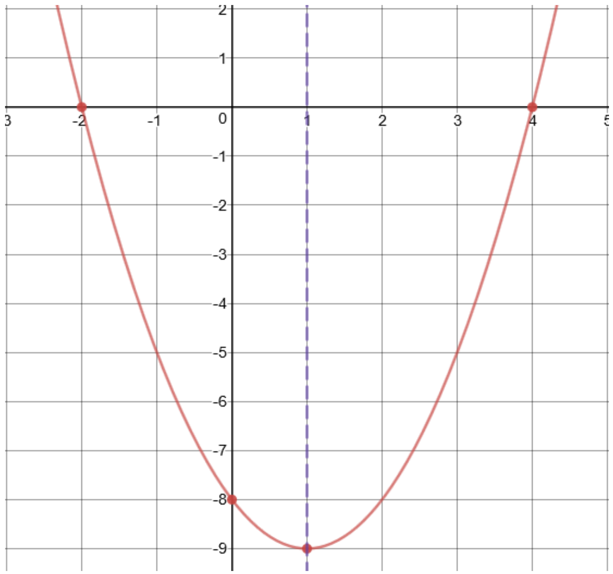
a) Graph the quadratic function by determine if the graph opens up or down and by finding its vertex, axis of symmetry, y-intercepts, and x-intercepts, if any.

b) Determine the domain and range of the function

c) Determine where the function is increasing and decreasing.

$$f(x) = x^2 - 2x - 8$$

where a=1 b=-2 and c=-8



a) i) Since a>0, the graph of the function opens up

ii) Vertex: $h = \frac{-b}{2a} = \frac{-(-2)}{2(1)} = 1;\ k = f\left(\frac{-b}{2a}\right) = (1)^2 - 2(1) - 8 = -9$

The vertex is (1,-9)

iii) Axis of symmetry x=h, x=1

iv) y-intercepts: set x=0 and solve

$f(0) = 0^2 - 2(0) - 8 = -8$

v) x-intercepts: set y=0 and solve for x

$$0 = x^2 - 2x - 8$$

f can solve by factoring, completing the square or use the quadratic formula

$$0 = (x - 4)(x + 2)$$

$$x = 4\ and\ x = -2$$

b) Domain (x-values): All real numbers or $(-\infty, \infty)$

Range (y-values): $y \geq -9$ or $[-9, \infty)$

c) Increasing $(-9, \infty)$

Decreasing: $(-\infty, -9)$

Determine the quadratic function (f(x)) for which the vertex is (-2,1) and contains the point (1,4). Write the function in vertex and standard form.

Vertex form: $f(x) = a(x-h)^2 + k$

Remember (h,k) is the vertex.

$$f(x) = a(x-(-2))^2 + 1$$

$$f(x) = a(x+2)^2 + 1$$

To figure out leading coefficient a, we plug in the point (1,4) and solve for a.

$$4 = a(1+2)^2 + 1$$

Add 1 and 2

$$4 = a(3)^2 + 1$$

$$4 = a(9) + 1$$

Subtract 1

$$3 = 9a$$

Divide by 9 and simplify

$$\frac{3}{9} = a$$

$$\frac{1}{3} = a$$

The vertex form of the quadratic formula is $f(x) = \frac{1}{3}(x+2)^2 + 1$

To get into the standard form: rewrite the vertex form like $f(x) = a(x-h)(x-h) + k$

$$f(x) = \frac{1}{3}(x+2)(x+2) + 1$$

FOIL (x+2)(x+2)

$$f(x) = \frac{1}{3}(x^2 + 4x + 4) + 1$$

$$f(x) = \frac{1}{3}x^2 + \frac{4}{3}x + \frac{4}{3} + 1$$

$$f(x) = \frac{1}{3}x^2 + \frac{4}{3}x + \frac{7}{3}$$

Solve the quadratic inequality
$x^2 < 2x + 6$

1) Bring everything to one side (the side with the square)
$x^2 < 2x + 6$

$$x^2 - 2x - 6 < 0$$

2) Solve the inequality as though the inequality sign was an equal sign.

$$x^2 - 2x - 6 = 0$$

Using the quadratic formula $x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{-(-2) \pm \sqrt{(-2)^2-4(1)(-6)}}{2(1)} = \frac{2 \pm \sqrt{4-(-24)}}{2} = \frac{2 \pm \sqrt{28}}{2} = \frac{2 \pm \sqrt{4}\sqrt{7}}{2} = \frac{2 \pm 2\sqrt{7}}{2} = 1 \pm \sqrt{7}$

(These values are approximately -1.6, 3.6

3) Use a table to determine the correct intervals

Interval	$(-\infty, 1-\sqrt{7})$	$(1-\sqrt{7},\ 1+\sqrt{7})$	$(1+\sqrt{7}, \infty)$
Test Number	-2	0	4
Value of f	$(-2)^2 - 2(-2) - 6 = 2$	$(0)^2 - 2(0) - 6 = -6$	$(4)^2 - 2(4) - 6 = 2$
Conclusion	Positive	Negative	Positive

Since we want to know $x^2 - 2x - 6 < 0$, i.e. negative values, we need to look to see where the conclusion is negative.

Thus the answer is $(1-\sqrt{7},\ 1+\sqrt{7})$.

Find the complex zeros of quadratic function.

$$f(x) = 2x^2 + 3x + 9$$

1) Set the function to equal to zero

$$0 = 2x^2 + 3x + 9$$

2) Solve using the quadratic formula (or by completing the square)

a=2 b=3 c=9

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a} = \frac{-3 \pm \sqrt{3^2 - 4(2)(9)}}{2(2)} = \frac{-3 \pm \sqrt{9 - 72}}{4} =$$

$$\frac{-3 \pm \sqrt{-63}}{4} = \frac{-3 \pm \sqrt{-1}\sqrt{9}\sqrt{7}}{4} = \frac{-3 \pm i3\sqrt{7}}{4} = \frac{-3 \pm 3i\sqrt{7}}{4}$$

Solve the absolute value equation

$$2|2x - 4| - 6 = 8$$

Get the absolute value by itself

$$2|2x - 4| - 6 = 8$$
$$2|2x - 4| = 14$$
$$|2x - 4| = 7$$

Use the fact that $|u| = a$ is equivalent to $u = -a \; or \; u = a$ and solve for x.

$$2x - 4 = -7$$
$$2x = -3$$
$$x = \frac{-3}{2}$$

$$2x - 4 = 7$$
$$2x = 11$$
$$x = \frac{11}{2}$$

Then $x = \frac{-3}{2}$ and $x = \frac{11}{2}$

Solve $|x-6|+4 \leq 10$

Get the absolute value by itself

$$|x-6|+4 \leq 10$$

$$|x-6| \leq 6$$

Use the fact that $|u| \leq a$ is equivalent to $-a \leq u \leq a$ and solve for x.

$$-6 \leq x-6 \leq 6$$

$$\text{add 6 to both sides}$$

$$0 \leq x \leq 12$$

The price p (in dollars) and the quantity x sold of t-shirts obeys the demand equation:

$$x = 1400 - 70p$$

a) Find the model that expresses the revenue R as a function of the price p.
b) What unit price should be used to maximize revenue?
c) If this price is charged (price from part b), what is the maximum revenue?
d) How many t-shirts are sold at this price?

a) In general, the revenue is R=xp

For R(p) we need everything needs to be in terms of p. We substitute the demand function into the revenue equation.

$$R(p) = (1400 - 70p)p$$
$$R(p) = 1400p - 70p^2$$
$$R(p) = -70p^2 + 1400p$$

The revenue function is $R(p) = -70p^2 + 1400p$

b) Remember to find the unit price (p) that results in maximum revenue is to find the h from the vertex.

$$h = p = \frac{-b}{2a} = \frac{-1400}{2(-70)} = \frac{-1400}{-140} = 10$$

The unit price is \$10

c) Substitute the unit price into the revenue function.

$$R(10) = -70(10)^2 + 1400(10) = 7000$$

The maximum revenue is \$7000

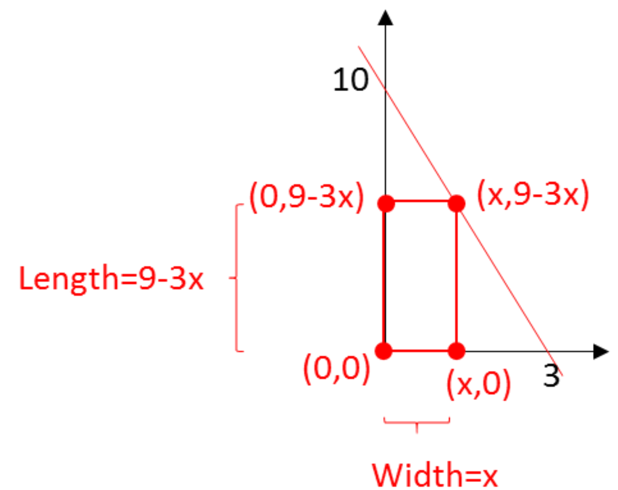
d) To find how many t-shirts were sold at \$10, substitute 10 into the demand equation.

$$x = 1400 - 70(10) = 700$$

700 shirts were sold at \$10.

A rectangle has one vertex on the line y=9-3x, x>0, one at the origin, one on the positive x-axis, and one on the positive-x-axis. Find the largest area A than can be enclosed by the rectangle.

1) Draw the rectangle (shown to the right)



2) Area=length x width
Length=9-3x
Width=x

$$A(x) = (9 - 3x)x$$
$$A(x) = 9x - 3x^2$$
$$A(x) = -3x^2 + 9x$$

3) Largest area means maximum area
To find the maximum area, find the vertex.

$$h = \frac{-b}{2a} = \frac{-9}{2(-3)} = \frac{-9}{-6} = \frac{3}{2}$$

$\frac{3}{2}$ is the x-value not the area.

$$A\left(\frac{-b}{2a}\right) = A\left(\frac{3}{2}\right) = -3\left(\frac{3}{2}\right)^2 + 9\left(\frac{3}{2}\right) = \frac{27}{4} = 6.75$$

The largest area is 6.75.