College Algebra

Chapter 4

Note:

This review is composed of questions similar to those in the chapter review at the end of chapter 4. This review is meant to highlight basic concepts from chapter 4. It does not cover all concepts presented by your instructor. Refer back to your notes, unit objectives, handouts, etc. to further prepare for your exam.

For $f(x) = \sqrt{x+4}$ and $g(x) = x^2 + 4$ find:

$$(f \circ g)(3)$$
$$(g \circ f)(-1)$$
$$(f \circ f)(0)$$
$$(g \circ g)(4)$$

$(f \circ g)(3)$

i) Find g(3): $g(3) = (3)^2$+4=13

ii) Find f(13): $f(13) = \sqrt{13+4}$ =$\sqrt{17}$

$(f \circ g)(3) = \sqrt{17}$

$(g \circ f)(-1)$

i) find f(-1): $f(-1) = \sqrt{-1+4}$ =$\sqrt{3}$

ii) find $g(\sqrt{3})$: $g(\sqrt{3})$=$(\sqrt{3})^2$+4=3+4=7

$(g \circ f)(-1)$=7

$(f \circ f)(0)$

i) find f(0): $\mathrm{f}(0) = \sqrt{0+4} = \sqrt{4} = 2$

ii) find f(2): $f(2) = \sqrt{2+4} = \sqrt{6}$

$(f \circ f)(0)$=$\sqrt{6}$

$(g \circ g)(4)$

i) Find g(4): $g(4) = 4^2 + 4 = 20$

ii) Find g(20): $g(20) = 20^2 + 4 = 404$

For $f(x) = \frac{x+2}{x-1}$ and $g(x) = \frac{1}{3x}$ find: $f \circ g$ and $g \circ g$. State the domain for each composite function.

$f \circ g$

$$f\left(\frac{1}{3x}\right) = \frac{\frac{1}{3x} + 2}{\frac{1}{3x} - 1}$$

Find the common denominator and multiply the function by $\frac{common\ denominator}{common\ denominator}$

$$f\left(\frac{1}{3x}\right) = \frac{\frac{1}{3x} + 2}{\frac{1}{3x} - 1} \cdot \frac{3x}{3x}$$

Multiple 3x through

$$f\left(\frac{1}{3x}\right) = \frac{1 + 6x}{1 - 3x}$$

$$(f \circ g)(x) = \frac{1 + 6x}{1 - 3x}$$

Domain: Find the domain of the result and the domain of the inside function.

Domain of the result ($\frac{1+6x}{1-3x}$) is $\{x|x \neq \frac{1}{3}\}$. The domain of the inside function (g). $\{x|x \neq 0\}$.

Combining these the domain of $f \circ g$ is $\{x|x \neq \frac{1}{3}, \mathrm{x} \neq 0\}$.

$g \circ g$

$$g\left(\frac{1}{3x}\right) = \frac{1}{3\left(\frac{1}{3x}\right)}$$

Simplify –threes cancel out

$$g\left(\frac{1}{3x}\right) = \frac{1}{\frac{1}{x}}$$

Rewrite

$$g\left(\frac{1}{3x}\right) = \frac{1}{1} \div \frac{1}{x}$$

Flip to make multiplication

$$g\left(\frac{1}{3x}\right) = \frac{1}{1} \cdot x = x$$

$$(g \circ g)(x) = x$$

Domain: Find the domain of the result and the domain of the inside function.

Domain of the result (x) is all real numbers The domain of the inside function (g). $\{x|x \neq 0\}$.

Combining these the domain of $g \circ g$ is $\{x|\ \mathrm{x} \neq 0\}$.

Find the inverse of $f(x) = \sqrt{x-5}$. The function is one-to-one.

$$f(x) = \sqrt{x-5}$$
$$y = \sqrt{x-5}$$
Switch x and y
$$x = \sqrt{y-5}$$
Solve for y
$$x^2 = y - 5$$
$$x^2 + 5 = y$$
$$f^{-1}(\mathrm{x}) = x^2 + 5$$

If $g(x) = \log_4 x$, evaluate $g(\frac{1}{16})$ and $g(64)$ by hand.

$$\boldsymbol{g\left(\frac{1}{16}\right)}$$

$$g\left(\frac{1}{16}\right) = \log_4\left(\frac{1}{16}\right)$$

Rewrite 16 in terms of the base of the logarithm (4)

$$g\left(\frac{1}{16}\right) = \log_4\left(\frac{1}{4^2}\right)$$

Use $\frac{1}{x^n} = x^{-n}$

$$g\left(\frac{1}{16}\right) = \log_4(4^{-2})$$

Use $\log_x x^n = n$

$$g\left(\frac{1}{16}\right) = -2$$

$$\boldsymbol{g(64)}$$

$$g(64) = \log_4(64)$$

Rewrite 64 in terms of the base of the logarithm (4)

$$g(64) = \log_4(4^3)$$

Use $\log_x x^n = n$

$$g(64) = 3$$

Find the domain of $h(x) = \log(\frac{3x}{x+1})$

1. Set the inside of the logarithm to be greater than zero and solve

$$\frac{3x}{x+1} > 0$$
$$3x > 0$$
$$x > 0$$

2. Set the denominator equal to zero and solve. x cannot be this value

$$x + 1 = 0$$
$$x = -1$$
$$x \neq -1$$

3. Combine the results from 1 and 2

Domain is $\{x | x > 0, x \neq -1\}$

Write the expression as the sum and/or difference of logarithms. Express powers as factors.

$$\log_3 \frac{3x^2}{wy}$$

Use $\log xy = \log x + \log y$ and $\log \frac{x}{y} = \log x - \log y$

$$\log_3 \frac{3x^2}{wy} = \log_3 3 + \log_3 x^2 - (\log_3 w + \log_3 y)$$

Simplify (remember $\log_a a = 1$

$$1 + \log_3 x^2 - \log_3 w - \log_3 y$$

Use $\log x^m = m \log x$

$$\log_3 \frac{3x^2}{wy} = 1 + 2\log_3 x - \log_3 w - \log_3 y$$

Write the expression as a single logarithm

$$\ln\left(\frac{x+2}{x}\right) + \ln\left(\frac{x}{x-2}\right) - \ln(x^2-4)$$

Use $\log xy = \log x + \log y$ and $\log\frac{x}{y} = \log x - \log y$

$$\ln\left(\left(\frac{x+2}{x}\right)\left(\frac{x}{x-2}\right)\right) - \ln(x^2-4)$$

$$\ln\left(\frac{\left(\frac{x+2}{x}\right)\left(\frac{x}{x-2}\right)}{x^2-4}\right)$$

Simplify

$$\ln\left(\frac{\left(\frac{x+2}{x-2}\right)}{x^2-4}\right)$$

$$\ln\left(\frac{\left(\frac{x+2}{x-2}\right)}{(x-2)(x+2)}\right) = \ln\left(\frac{x+2}{x-2} \div ((x-2)(x+2))\right)$$

$$\ln\left(\frac{x+2}{x-2} \cdot \frac{1}{(x-2)(x+2)}\right)$$

Cross out the x+2 from the numerator and the denominator

$$\ln\left(\frac{1}{(x-2)(x-2)}\right) = \ln\left(\frac{1}{(x-2)^2}\right)$$

For each function f

a) find the domain of f

b) Graph f

c) From the graph, determine the range and any asymptotes of f

d) find f^{-1}

e) Find the domain and range of f^{-1}

f) graph f^{-1}

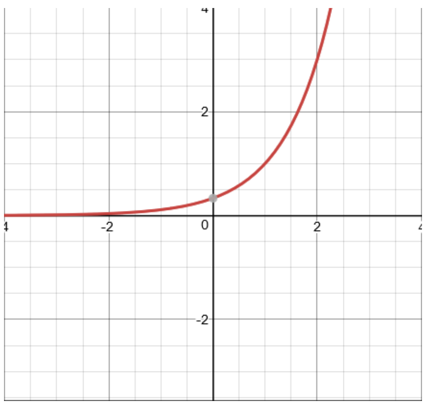
i) $f(x) = 3^{x-1}$

ii) $f(x) = \ln(x+2)$

$$\boldsymbol{f(x) = 3^{x-1}}$$

a) Domain is all real numbers

b)



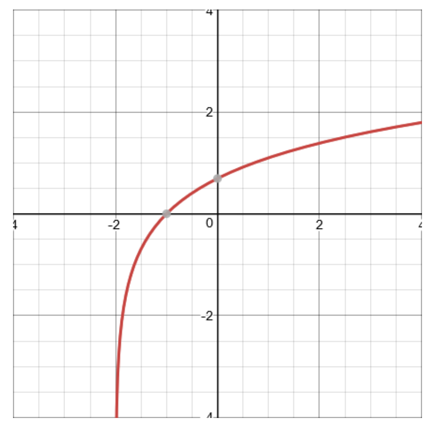
c) The range is $\{y|y > 0\}$

$$\boldsymbol{f(x) = \ln(x+2)}$$

a) Domain

$$x + 2 > 0$$
$$x > -2$$
$$\{x|x > -2\}$$

b)



c) The range is all real numbers

For each function f

a) find the domain of f

b) Graph f

c) From the graph, determine the range and any asymptotes of f

d) find f^{-1}

e) Find the domain and range of f^{-1}

f) graph f^{-1}

i) $f(x) = 3^{x-1}$

ii) $f(x) = \ln(x+2)$

$\boldsymbol{f(x) = 3^{x-1}}$

d. $y = 3^{x-1}$

$x = 3^{y-1}$

$\log_3 x = y - 1$

$\log_3 x + 1 = y$

$f^{-1}(\mathrm{x}) = \log_3 x + 1$

$\boldsymbol{f(x) = \ln(x+2)}$

d. $y = \ln(x+2)$

$x = \ln(y+2)$

$e^x = y + 2$

$e^x - 2 = y$

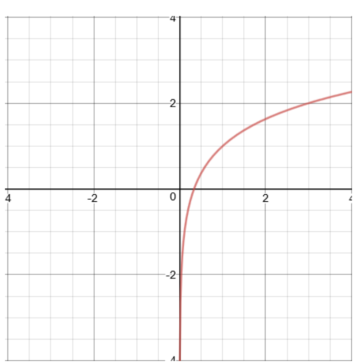
$f^{-1}(\mathrm{x}) = e^x - 2$

Remember Domain of f = range of f^{-1} and the range of f = domain of f^{-1}

e. Domain: $\{x|x > 0\}$

Range: All real numbers

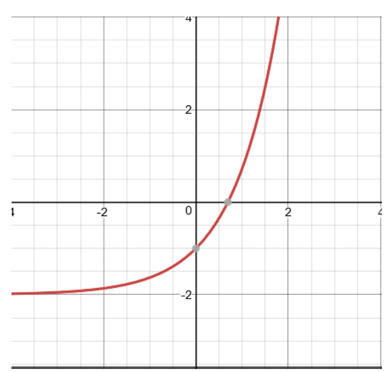
f.



e. Domain: all real numbers

Range: $\{y|y > -2\}$

f.



Solve

a) $4^{x+2} = 16^x$ b)$3^x = 6^{x+1}$ c) $9^x - 6 \cdot 3^x + 5 = 0$

$\mathbf{4^{x+2} = 16^x}$	$\mathbf{3^x = 6^{x+1}}$	$\mathbf{9^x - 6 \cdot 3^x + 5 = 0}$
Write each with the same base $4^{x+2} = (4^2)^x$ $4^{x+2} = 4^{2x}$ Since the bases are the same, we can cross them out $x + 2 = 2x$ Solve for x $2 = x$	Take the Log of both sides $Log\ 3^x = Log\ 6^{x+1}$ Use $\log x^m = m \log x$ $x \log 3 = (x + 1) \log 6$ Distribute $\log 6$ $x \log 3 = x \log 6 + \log 6$ Bring everything with an x to the same side $x \log 3 - x \log 6 = \log 6$ Factor out x $x\,(\log 3 - \log 6) = \log 6$ Solve for x $x = \frac{\log 3 - \log 6}{\log 6}$	Get common exponential $(3^x)^2 - 6 \cdot 3^x + 5 = 0$ Use u substitution for 3^x $u^2 - 6u + 5 = 0$ Solve for u (factor) $(u - 5)(u - 1) = 0$ $u = 5\ or\ u = 1$ Use values for u to solve for 3^x $5 = 3^x\ or\ 1 = 3^x$ For each equation, take the log on both sides to solve for x $\log_3 5 = x\ or\ \log_3 1 = x$ $x \approx 1.465\ or\ x = 0$

Solve

a) $\ln\sqrt{x+2} = 4$

b) $\log(x+2) + \log(x+1) = 1$

$\boldsymbol{ln\sqrt{x+2} = 4}$

Write in exponential form

$e^4 = \sqrt{x+2}$

Square both sides

$(e^4)^2 = (\sqrt{x+2})^2$

Remember $(x^m)^n = x^{mn}$

$e^8 = x + 2$

Solve for x

$e^8 - 2 = x$

$\boldsymbol{log(x+2) + log(x+1) = 1}$

Combine Logs

$\log((x+2)(x+1)) = 1$

FOIL

$\log(x^2 + x + 2x + 2) = 1$

$\log(x^2 + 3x + 2) = 1$

Write in exponential form

$x^2 + 3x + 2 = 10^1$

$x^2 + 3x + 2 = 10$

$x^2 + 3x - 8 = 0$

Solve using the quadratic formula (not shown)

$x = \frac{-3 \pm \sqrt{41}}{2}$

However only $x = \frac{-3+\sqrt{41}}{2}$ works

A child's aunt wishes to purchase a bond that matures in 18 years to be used for his college education. The bond pays 6% interest compounded semiannually. How much should they pay so the bond is worth $150,000 at maturity.

Use $A = P(1 + \frac{r}{n})^{nt}$

P=? A=150000 t=18 n=2 (semiannually) r=.06

$$150000 = P(1 + \frac{.06}{2})^{2 \cdot 18}$$

Solve for P

$$\frac{150000}{(1 + \frac{.06}{2})^{2 \cdot 18}} = P$$

$$P \approx 4045.31$$

A pan is removed from an oven where the temperature is $425°F$ and placed in a room whose temperature is $68°F$. After 10 minutes, the temperature of the pan is $375°F$. How long will it be until the temperature is $200°F$. Round k to 3 decimal places

Using Newton's Law of cooling ($u(t) = T + (u_0 - T)e^{kt}$)
T=temperature of the room =68
u_0=the initial temperature of the pan=425
k=negative constant currently unknown

1) Find K. Let $u(t)$=375 when t=10

$$375 = 68 + (425 - 68)e^{k10}$$

Combine like terms

$$375 = 68 + 357e^{k10}$$

Subtract 68 from both sides

$$307 = 357e^{k10}$$

Divide by 357

$$\frac{307}{357} = e^{k10}$$

Rewrite in logarithmic form

$$\ln\frac{307}{357} = k10$$

Divide by 10

$$k = \frac{\ln\frac{307}{357}}{10} \approx -.015$$

2) Substitute the known values of k, let $u(t)$=200, and solve for t

$$200 = 68 + (425 - 68)e^{-.015t}$$

Combine like terms

$$200 = 68 + (357)e^{-.015t}$$

Subtract 68 from both sides

$$132 = 357e^{-.015t}$$

Divide by 357

$$\frac{132}{357} = e^{-.015t}$$

Rewrite in logarithmic form

$$\ln\frac{132}{357} = -.015t$$

Divide by -.015

$$t = \frac{\ln\frac{132}{357}}{-.015} \approx 66.3\ minutes$$