College Algebra

Chapter 5

Note:

This review is composed of questions similar to those in the chapter review at the end of chapter 5. This review is meant to highlight basic concepts from chapter 5. It does not cover all concepts presented by your instructor. Refer back to your notes, unit objectives, handouts, etc. to further prepare for your exam.

Identify the equations as a parabola, ellipse, or hyperbola. If
a) parabola, find the vertex, focus, and directrix
b) ellipse, find the center, vertices, and foci
c) hyperbola, find the center, vertices, foci, and asymptotes

$$\boldsymbol{y^2 = -64x}$$

This is a *parabola* opening left

Standard form is $(y-k)^2 = 4a(x-h)$

$$(y-k)^2 = 4a(x-h)$$
$$y^2 = -64x$$

4a=-64

a=-16

Vertex: (h,k)=(0,0)

Focus: Since y is squared (h+a,k)=(-16,0)

Directrix: x=h-a so x=16

Identify the equation as a parabola, ellipse, or hyperbola. Rewrite the in standard form.

$$\mathbf{6x^2 + 3y^2 - 48x + 18y = 3}$$

This is an *ellipse*

Rewrite in standard form by completing the square

1. Write the x's by each other and the y's by each other

$$6x^2 - 48x + 3y^2 + 18y = 3$$

2. Factor out what is in front of the x^2 and y^2

$$6(x^2 - 8x) + 3(y^2 + 6y) = 3$$

3. Take half of the term that has x or y, square it, add to both sides. When adding the number to the right hand side, you need to multiply it by what's in front of the parenthesis (the 6 or 3).

$$6(x^2 - 8x + \underbrace{16}_{\left(\frac{-8}{2}\right)^2 = 16}) + 3(y^2 + 6y + \underbrace{9}_{\left(\frac{6}{2}\right)^2 = 9}) = 3 + \underbrace{96}_{16 \cdot 6} + \underbrace{27}_{9 \cdot 3}$$

3. Factor the Left hand side and find the sum on the right hand side

$$6(x-4)^2 + 3(y+3)^2 = 126$$

4. Divide by 126 and simplify to have the ellipse in standard form

$$\frac{(x-4)^2}{21} + \frac{(y+3)^2}{42} = 1$$

Identify the equation as a parabola, ellipse, or hyperbola. If
a) parabola, find the vertex, focus, and directrix
b) ellipse, find the center, vertices, and foci
c) hyperbola, find the center, vertices, foci, and asymptotes

$$\frac{x^2}{36} + \frac{y^2}{9} = 1$$

This is an *ellipse*

Standard form is $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Center (h,k) =(0,0)

a=6 b=3

Vertices:

a is under the x so the vertices are

$$(h \pm a, k) = (0 \pm 6,0) = (6,0) \text{ and } (-6,0)$$

Foci:

Find c using $b^2 = a^2 - c^2$

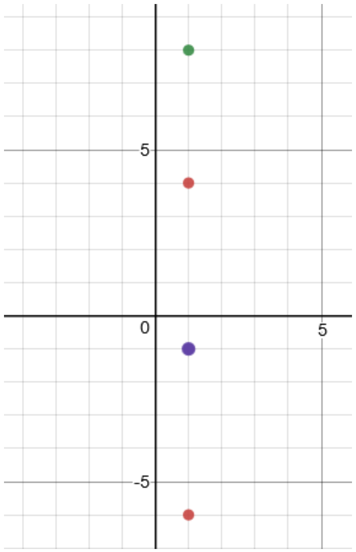
$$c^2 = 9 = 36 - c^2 = 27$$

$$c^2 = 27\ c = \sqrt{27} = \sqrt{9}\sqrt{3} = 3\sqrt{3}$$

a is under the x so the foci is are

$$(h \pm c, k) = \left(0 \pm 3\sqrt{3}, 0\right)$$

Find the equation of the conic described. Graph the equation.
Ellipse: Foci (1,4) and (1,-6); vertex at (1,8)



1. Plot the known points
The Center is between the Foci which is (1,-1)
2. a is the distance from the vertex to the center
a=9
Thus the other vertex is (1,-1-9)=(1,-10)
3. Find b using $b^2 = a^2 - c^2$ (c is the distance from the foci to the center =5)

$$b^2 = 9^2 - 5^2$$

$$b^2 = 56 \; b = \sqrt{56} = 2\sqrt{14}$$

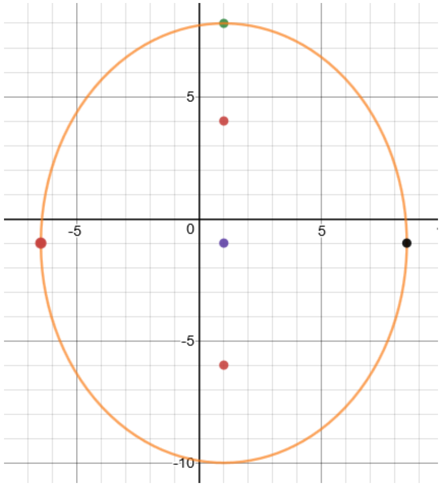
The minor endpoints are $(1 \pm 2\sqrt{14}, -1)$
4. The standard form is

$$\frac{(x-h)^2}{b^2} + \frac{(y-k)^2}{a^2} = 1$$

(the a is under the y because the vertices and foci are parallel to the y-axis)

$$\frac{(x-1)^2}{56} + \frac{(y+1)^2}{81} = 1$$

5. To Graph, plot the minor endpoints and connect the vertices and the minor endpoints.



Identify the equation as a parabola, ellipse, or hyperbola. Rewrite the in standard form.

$$\boldsymbol{x^2 + 6x = 3y}$$

This is a parabola opening up

Standard form is $(x-h)^2 = 4a(y-k)$

Rewrite in standard form by completing the square

$$x^2 + 6x = 3y$$

Take half of 6 (3), square it (9) and add to both sides

$$x^2 + 6x + 9 = 3y + 9$$

Factor Left Hand Side

$$(x+3)^2 = 3y + 9$$

Factor out 3 on the right hand side

$$(x+3)^2 = 3(y+3)$$

$$4a = 3 \text{ so } a = \frac{3}{4}$$

Vertex: (h,k)=(-3,-3)

Focus: (k,k+a)=$\left(-3, \frac{-9}{4}\right)$

Directrix: y=k-a =$-\frac{15}{4}$

Identify the equation as a parabola, ellipse, or hyperbola. If
a) parabola, find the vertex, focus, and Directrix
b) ellipse, find the center, vertices, and foci
c) hyperbola, find the center, vertices, foci, and asymptotes

$$\boldsymbol{y^2 - \frac{(x-1)^2}{16} = 1}$$

This is a *hyperbola*.
The standard form when the y is first is

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Center (h,k) is (1,0)
a=1 b=4
Vertices $(h, k \pm a) = (1, \pm 1) = (1, -1)$ and $(1,1)$
Foci: Find c using $b^2 = c^2 - a^2$

$$16 = c^2 - 1$$

$$c^2 = 17 \; c = \sqrt{17}$$

Foci are at $(h, k \pm c) = \left(1, \pm\sqrt{17}\right)$
Asymptotes: $y - k = \pm \frac{a}{b}(x - h)$

$$y - 0 = \pm \frac{1}{16}(x - 1)$$

$$y = \frac{1}{16}x - \frac{1}{16} \; and \; y = -\frac{1}{16}x + \frac{1}{16}$$

Identify the equation as a parabola, ellipse, or hyperbola. Rewrite the in standard form.

$$\boldsymbol{x^2 - 4y^2 + 4x + 10y = 29}$$

This is an *hyperbola*.

Rewrite in standard form by completing the square

1. Write the x's by each other and the y's by each other

$$x^2 + 4x - 4y^2 + 10y = 29$$

2. Factor out what is in front of the x^2 and y^2. Be sure to factor out the negative with the y's.

$$(x^2+4x) - 4(y^2-\frac{10}{4}y) = 29$$

$$(x^2+4x) - 4(y^2-\frac{5}{2}y) = 29$$

3. Take half of the term that has x or y, square it, add to both sides. When adding the number to the right hand side, you need to multiply it by what's in front of the parenthesis (the 1 or -4).

$$(x^2+4x+4) - 4(y^2-\frac{5}{2}y+\frac{25}{4}) = 29 + 4 - 25$$

$$\left(\frac{-4}{2}\right)^2 = 4 \qquad \left(\frac{-5/2}{2}\right)^2 = \frac{25}{4} \qquad 4 \cdot 1 \qquad \frac{25}{4} \cdot -4$$

3. Factor the two parts on left hand side and find the sum on the right hand side

$$(x+2)^2 - 4(y-\frac{5}{4})^2 = 8$$

4. Divide by 126 and simplify to have the hyperbola in standard form

$$\frac{(x+2)^2}{8} - \frac{\left(y-\frac{5}{4}\right)^2}{2} = 1$$

Find the equation of the conic described. Graph the equation.
Center at (2,-1) a=3 c=5; traverse axis parallel to the y-axis.

This is a hyperbola because it has a traverse axis.

1. Find b using $b^2 = c^2 - a^2$

$$b^2 = 5^2 - 3^2$$

$$b^2 = 16 \quad b = 4$$

2. A hyperbola with traverse axis parallel to the y-axis has the standard form of

$$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$$

Put what you know into the standard form of a hyperbola.

$$\frac{(y+1)^2}{9} - \frac{(x-2)^2}{16} = 1$$

Graphing is on the next slide

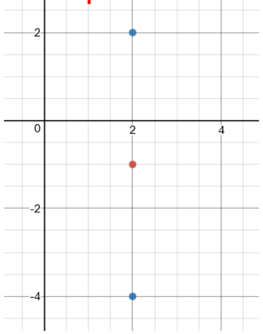
Find the equation of the conic described. Graph the equation.
Center at (2,-1) a=3 c=5; traverse axis parallel to the y-axis.

To Graph

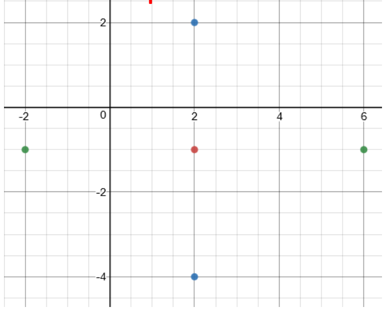
$$\frac{(y+1)^2}{9}-\frac{(x-2)^2}{16}=1$$

1. Plot the Center (2,-1)
2. From the center go up and down 3 (a) to get the vertices (2,-4) and (2,2)
3. Find the other endpoints for the box by going left and right by 4 (b). These points are (-2,-1) and (6,-1).
4. Using the vertices and the other endpoints, draw the box.
5. Draw the asymptotes. Remember the asymptotes go through the corners of the box.
6. Erase the box and plot the Foci. The foci is up/down c from the center (2,-6) and (2,4)
7. Draw the Hyperbola (plot more points to make the graph more accurate.

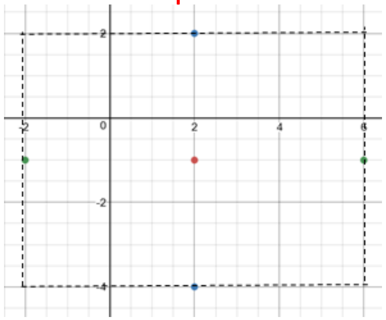
Step 1 and 2



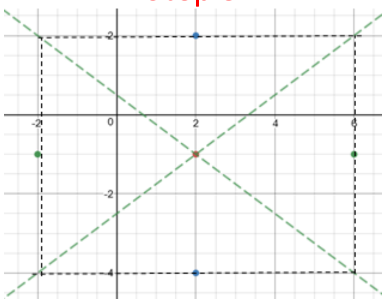
Step 3



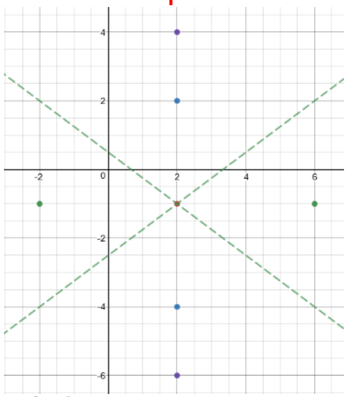
Step 4



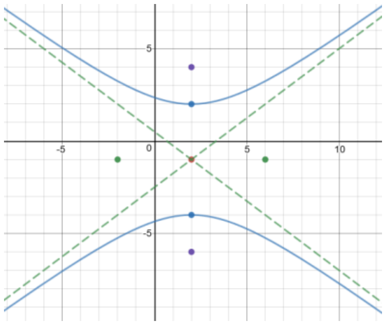
Step 5



Step 6



Step 7



A bridge is built in the shape of a semielliptical arch. The bridge has a span of 30 feet and a maximum height of 10. What is the height of the bridge 5 feet from the center

1. Draw a picture of the bridge
The entire span is 30 feet so that is 15 from the center on either side.
a=15
b=height=10

2. Put what we know into the formula for an ellipse.

$$\frac{x^2}{15^2} + \frac{y^2}{10^2} = 1$$
$$\frac{x^2}{225} + \frac{y^2}{100} = 1$$

3.To find out what the height is at 5 feet, substitute 5 for x and solve for y.

$$\frac{5^2}{225} + \frac{y^2}{100} = 1$$

Solving for y we get

$$y \approx 9.428$$