Point Estimate

$\hat{p} = \frac{x}{n}$ — $\hat{p}$ is the point estimate for the population proportion, x is the number of successes, n is the sample size

$\hat{p} = \frac{Upper\ Bound + Lower\ Bound}{2}$ for the population proportion

$\bar{x} = \frac{Upper\ Bound + Lower\ Bound}{2}$ for the population mean

$E = \frac{Upper\ Bound - Lower\ Bound}{2}$ -> E is the margin of error (can be used for either)

A survey was given to 1500 students in a school to find the proportion of students who are interested in taking a musical course. Determine the point estimate, margin of error, and number of students who responded they would be interested in a musical course, based on the given upper and lower bounds.

Lower bound = .682 Upper bound = .746

$$\hat{p} = \frac{Upper\ Bound + Lower\ Bound}{2} = \frac{.746 + .682}{2} = .714$$

$$E = \frac{Upper\ Bound - Lower\ Bound}{2} = \frac{.746 - .682}{2} = 0.032$$

$\hat{p} = \frac{x}{n}$ -> $x = \hat{p} \cdot n = .714 \cdot 1500 = 1071\ students\ interested$

Confidence Intervals

Population proportion:	$\hat{p} \pm z_{a/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$	->	Use z-table
Population mean:	$\bar{x} \pm t_{a/2} \cdot \frac{s}{\sqrt{n}}$	->	Use t-table
Population variance:	$\frac{(n-1)s^2}{\chi^2_{a/2}}, \frac{(n-1)s^2}{\chi^2_{1-a/2}}$	->	Use chi-square table
Population standard deviation:	$\sqrt{\frac{(n-1)s^2}{\chi^2_{a/2}}}, \sqrt{\frac{(n-1)s^2}{\chi^2_{1-a/2}}}$	->	Use chi-square table

$$\frac{a}{2} = \frac{1 - Confidence\ Level}{2}$$

Common critical values for the <u>POPULATION PROPORTION ONLY</u>:

Confidence Level	Critical Value (Z-Score)
90%	1.645
95%	1.96
98%	2.33
99%	2.575

For the population mean, variance, or standard deviation, the degrees of freedom (df), will also be needed to find the critical value.

$df = n - 1$

Sample Size

Always round up to the next integer

Population proportion: $n = \hat{p}(1-\hat{p})(\frac{z_{a/2}}{E})^2$

**If there are no prior estimates, then use $\hat{p} = .5$

Population mean: $n = (\frac{z_{a/2} \cdot s}{E})^2$

Examples: A survey of 500 airline passengers found that 338 were satisfied with the service they received from the flight attendants. Calculate and interpret a 95% confidence interval for the proportion of passengers who are satisfied with the service from flight attendants.

- Because we are looking for a population proportion, first we need to find the point estimate, and then we will use the z-table in our confidence interval for the critical value.

$$\hat{p} = \frac{x}{n} = \frac{338}{500} = .676$$

Now, we use $\hat{p} \pm z_{a/2} \cdot \sqrt{\frac{\hat{p}(1-\hat{p})}{n}}$, and use the z-table. For a 95% confidence interval, the critical value is 1.96

$$LB = .676 - 1.96 \cdot \sqrt{\frac{.676(1-.676)}{500}} = .635$$

$$UB = .676 - 1.96 \cdot \sqrt{\frac{.676(1-.676)}{500}} = .717$$

So a 95% confidence interval for the proportion of satisfied airline passengers is $(.635, .717)$.

We are 95% confident that the proportion of airline passengers who are satisfied with their service from flight attendants is between .635 and .717.

A study was conducted using 80 dancers in a dance competition, to determine how many hours they practice on average. The study found that on average, the dancers practiced 7.35 hours per week, and the sample standard deviation of the dancers surveyed was 1.28 hours. Calculate and interpret a 98% confidence interval for the mean number of hours practiced by the dancers at the competition.

- Because we are estimating the population mean, we need to find the point estimate, and we will use the t-table in our confidence interval for the critical value.

$\bar{x} = 7.35, s = 1.28$

$$\frac{a}{2} = \frac{1-.98}{2} = .01$$

Now we use $\bar{x} \pm t_{a/2} \cdot \frac{s}{\sqrt{n}}$, and use the t-table. The degrees of freedom here is 80-1, or 79. We will use the line for 80 degrees of freedom on the table. At df=80, and $\alpha/2 = .01$, we get a critical value of 2.374

$$LB = 7.35 - 2.374 \cdot \frac{1.28}{\sqrt{80}} = 7.010$$

$$UB = 7.35 + 2.374 \cdot \frac{1.28}{\sqrt{80}} = 7.690$$

Our 98% confidence interval is $(7.010, 7.690)$

We are 98% confident that the mean number of hours the dancers practice each week is between 7.010 and 7.690

A survey given to 25 incoming freshmen at a college in Illinois has shown that the average composite ACT score of an incoming freshman is 21.4. The standard deviation of these ACT scores for those freshmen is 5.78. Calculate and interpret a 90% confidence interval for the population standard deviation of composite ACT scores at that college in Illinois.

- Because we are estimating the population standard deviation, we will use the chi-square table. For the confidence interval, we will use $\sqrt{\frac{(n-1)s^2}{\chi^2_{a/2}}}, \sqrt{\frac{(n-1)s^2}{\chi^2_{1-a/2}}}$

$\frac{a}{2} = \frac{1-.90}{2} = .05, 1 - \frac{a}{2} = 1 - .05 = .95$

The degrees of freedom is 25-1, or 24. The two critical values then come from df=24, $a/2 = .05$, and df=24, $a = .95$. Those critical values are 36.415 and 13.848

$$LB = \sqrt{\frac{(25-1)5.78^2}{36.415}} = 4.69$$

$$UB = \sqrt{\frac{(25-1)5.78^2}{13.848}} = 7.61$$

Our 90% confidence interval is thusly $(4.69, 7.61)$

We are 90% confident that the population standard deviation of composite ACT scores at the college in Illinois is between 4.69 and 7.61.

Using the ACT data, compute and interpret a 99% confidence interval for the mean composite ACT score at the college in Illinois.

- Because we are estimating the population mean, we will need to use the t-table. The degrees of freedom is still 24. Now we will find $\alpha/2$.

$\frac{a}{2} = \frac{1-.99}{2} = .005$

With df=24, and $a = .005$, we look at the t-table, and get a critical value of 2.797

Now we use $\bar{x} \pm t_{a/2} \cdot \frac{s}{\sqrt{n}}$

$$LB = 21.4 - 2.797 \cdot \frac{5.78}{\sqrt{25}} = 18.167$$

$$UB = 21.4 + 2.797 \cdot \frac{5.78}{\sqrt{25}} = 24.633$$

The 99% confidence for the mean composite ACT score is $(18.167, 24.633)$

We are 99% confident that the population mean composite ACT score at the college in Illinois is between 18.167 and 24.633

A political firm wants to conduct a survey to gauge voter interest of a particular candidate in the state of Minnesota. They are looking for a 95% confidence level, and the margin of error desired on the survey is 2%. What is the sample size needed if the prior estimate of people in favor of this candidate is 41%?

- Because we are working with population proportion, we will use $n = \hat{p}(1 - \hat{p})(\frac{z_{a/2}}{E})^2$
- Also, $z_{a/2} = 1.96$ for 95% confidence

$$n = .41(1 - .41)\left(\frac{1.96}{.02}\right)^2 = 2323.21$$

So the political firm would need 2324 participants in the survey

What if there were no prior estimates?

- Without prior estimates, we use $\hat{p} = .5$

$$n = .5(1 - .5)\left(\frac{1.96}{.02}\right)^2 = 2401$$

With no prior estimates, the political firm would need 2401 participants for the survey