Deck of Cards Questions

- There are 52 cards in a standard deck of cards
- There are 4 of each card (4 Aces, 4 Kings, 4 Queens, etc.)
- There are 4 suits (Clubs, Hearts, Diamonds, and Spades) and there are 13 cards in each suit (Clubs/Spades are black, Hearts/Diamonds are red)
- Without replacement means the card IS NOT put back into the deck. With replacement means the card IS put back into the deck.

Examples:

What is the probability that when two cards are drawn from a deck of cards without replacement that both of them will be 8's?

$$P(Both\ are\ 8's) = P(First\ card\ is\ an\ 8) \cdot P(Second\ card\ is\ an\ 8)$$

$$P(First\ card\ is\ an\ 8) = \frac{4}{52}$$

- There are three 8's left in the deck if one is pulled and not replaced, and 51 total cards remaining.

$$P(Second\ card\ is\ an\ 8) = \frac{3}{51}$$

$$P(Both\ are\ 8's) = \frac{4}{52} \cdot \frac{3}{51} = \frac{12}{2652} = \frac{1}{221} = .0045\ or\ .45\%$$

What is the probability that both cards drawn (without replacement) will be spades?

$$P(Both\ are\ spades) = P(First\ card\ is\ a\ spade) \cdot P(Second\ card\ is\ a\ spade)$$

$$P(First\ card\ is\ a\ spade) = \frac{13}{52}$$

- There are 12 spades left in the deck if one is pulled and not replaced, and 51 total cards remaining.

$$P(Second\ card\ is\ a\ spade) = \frac{12}{51}$$

$$P(Both\ are\ spades) = \frac{13}{52} \cdot \frac{12}{51} = \frac{156}{2652} = \frac{1}{17} = .0588\ or\ 5.88\%$$

What is the probability of drawing a red king and then a black 7 without replacement?

$$P(Red\ king\ then\ black\ 7) = P(Red\ king) \cdot P(Second\ card\ is\ a\ black\ seven)$$

- There are 4 of each card, so there are 2 red and 2 black of each card. This means we have 2 red kings in the deck, and 2 black 7's in the deck.

$$P(Red\ king) = \frac{2}{52}$$

*Even with a red king drawn first, there will still be 2 black 7's in the deck, but only 51 cards remaining.

$$P(Second\ card\ is\ a\ black\ seven) = \frac{2}{51}$$

$$P(Red\ king\ then\ black\ 7) = \frac{2}{52} \cdot \frac{2}{51} = \frac{4}{2652} = \frac{1}{663} = .0015\ or\ .15\%$$

What is the probability of being dealt a flush (5 cards of all the same suit) from the first 5 cards in a deck?

- The first card it does not matter what the suit is. Any of the suits can be drawn initially, as long as the next four cards are of the same suit as the original card.
- There are 13 of each suit in the deck, so after the first card is drawn, there are only 12 of that suit, then 11 left for the third card, 10 left for the fourth card, and 9 left for the final card. Also, there will be one less card total in the deck each time.

$$P(flush) = P(2nd\ is\ same\ suit) \cdot P(3rd\ is\ same\ suit) \cdot P(4th\ is\ same\ suit) \cdot P(5th\ is\ same\ suit)$$

$$P(flush) = \frac{12}{51} \cdot \frac{11}{50} \cdot \frac{10}{49} \cdot \frac{9}{48} = \frac{11880}{5997600} = \frac{33}{16660} = .00198\ or\ .198\%$$

What is the probability of drawing two face cards, and then 2 numbered cards, without replacement?

- There are 12 face cards (Kings, queens, and jacks) and there are 36 numbered cards (2's through 10's).
- After the first face card is drawn, there will be 11 face cards leftover, and 51 total cards remaining.

$P(2\ face\ cards) = \frac{12}{52} \cdot \frac{11}{51}$

- Now we only have 50 cards left in the deck, but all 36 of the numbered cards are still in there. After one is drawn, there are 35 numbered cards remaining of the 49 total cards that now remain.

$P(2\ numbered\ cards) = \frac{36}{50} \cdot \frac{35}{49}$

$P(2\ face\ cards\ then\ 2\ \#\ cards) = P(2\ face\ cards) \cdot P(2\ numbered\ cards)$

$P(2\ face\ cards\ then\ 2\ \#\ cards) = \frac{12}{52} \cdot \frac{11}{51} \cdot \frac{36}{50} \cdot \frac{35}{49} = \frac{166320}{6497400} = \frac{198}{7735} = .0256\ or\ 2.56\%$

What is the probability of drawing an Ace 3 times in a row with replacement?

$P(3\ Aces\ in\ a\ row) = P(Card\ 1\ is\ an\ Ace) \cdot P(Card\ 2\ is\ an\ Ace) \cdot P(Card\ 3\ is\ an\ Ace)$

- This time, we are replacing the card, which means there will always be 4 Aces in the deck, and always 52 total cards.

$P(Getting\ an\ Ace) = \frac{4}{52}$

$P(3\ Aces\ in\ a\ row) = \frac{4}{52} \cdot \frac{4}{52} \cdot \frac{4}{52} = \frac{1}{2197} = .000455 = .0455\%$