Five Number Summaries and Boxplots

The five parts of a five number summary are:

1. Minimum value
2. First Quartile (Q1)
3. Median value
4. Third Quartile (Q3)
5. Maximum value

*To be able to find the median and the first and third quartiles, the data must be in order from smallest to largest

To find the Median, find the middle number. Cross out one at a time, alternating from the beginning of the list of numbers with the end of the list. This is done in pairs. If one number is leftover, it is your median. If two numbers are leftover, find what is halfway between them to find the median.

To find the 1\textsuperscript{st} quartile, take the first half of the data, and do the same process that is done for finding the median. To find the 3\textsuperscript{rd} quartile, take the second half of the data and do the same process that is done for finding the median.

Using the Five Number Summary

The Interquartile Range (IQR) is $Q_3 - Q_1$

The upper and lower fences determine whether or not a data point is an outlier. If the data point is not between the upper and lower fence, the point is an outlier.

Lower Fence = $Q_1 - 1.5(IQR)$

Upper Fence = $Q_3 - 1.5(IQR)$

Below is a table of the final exam scores from a class of 40 statistics students. Use the table to answer the following questions.

<table>
<thead>
<tr>
<th>Statistics Final Exam Scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>86</td>
</tr>
<tr>
<td>84</td>
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</table>
What is the five number summary for this set of data?

First put the data in order:

54, 62, 67, 70, 72, 73, 73, 73, 73, 74, 77, 77, 78, 79, 79, 80, 81, 82, 83, 84, 84, 84, 84, 85, 85, 85, 85, 86, 86, 87, 88, 89, 90, 90, 91, 93, 95, 96, 98, 99

In the above example, the first half of the data is in red and the second half in blue. The numbers in green are the middle of each half, thus helping us find the first and third quartiles. Using that, we have:

1. Minimum = 54
2. First Quartile = 75.5
3. Median = 84
4. Third Quartile = 87.5
5. Maximum = 99

Find the interquartile range for the exam data

\[ Q3 = 87.5, \quad Q1 = 75.5 \rightarrow \text{IQR} = 87.5 - 75.5 = 12 \]

Find the upper and lower fences for the exam data, and determine if there are any outliers in the data set

Lower Fence = 75.5 - 1.5(12) = 57.5

Upper Fence = 87.5 + 1.5(12) = 105.5

There is one outlier -> the exam score of 54

Find the mean and the standard deviation of the exam data

\[ \text{Mean} = \frac{\text{Sum of all of the scores}}{40} = 81.75 \]

\[ \text{Standard Deviation} = \sqrt{\frac{\sum x^2 - (\sum x)^2}{n}} = 9.43 \]
Create a box and whisker plot of the exam data

Statistics Final Exam Scores

50  55  60  65  70  75  80  85  90  95  100