Rationalizing Denominators

Square roots:

- Multiply the numerator and denominator by the denominator. Then simplify.

Examples

Simplify the following expression: \( \frac{6}{\sqrt{7}} \)

\[
\frac{6}{\sqrt{7}} = \frac{6 \cdot \sqrt{7}}{\sqrt{7} \cdot \sqrt{7}}
\]

*Multiply both numerator and denominator by \( \sqrt{7} \)

\[
= \frac{6\sqrt{7}}{7}
\]

Because we cannot simplify any further, \( \frac{6\sqrt{7}}{7} \) is our final answer.

Simplify the following expression: \( \frac{10}{\sqrt{5}} \)

\[
\frac{10}{\sqrt{5}} = \frac{10 \cdot \sqrt{5}}{\sqrt{5} \cdot \sqrt{5}}
\]

*Multiply both numerator and denominator by \( \sqrt{5} \)

\[
= \frac{10\sqrt{5}}{5}
\]

*We see that we can simplify this fraction further by dividing the numerator and denominator each by 5

\[
= 2\sqrt{5}
\]

Because we cannot simplify any further, \( 2\sqrt{5} \) is our final answer.

Simplify the following expression: \( \frac{\sqrt{6}}{\sqrt{15}} \)

\[
\frac{\sqrt{6}}{\sqrt{15}} = \frac{\sqrt{6} \cdot \sqrt{15}}{\sqrt{15} \cdot \sqrt{15}}
\]

*Multiply both numerator and denominator by \( \sqrt{15} \)

\[
= \frac{\sqrt{90}}{15}
\]

*Now we check to see whether \( \sqrt{90} \) can be simplified, which we can do

\[
= \frac{\sqrt{9\cdot10}}{15}
\]

*Split up \( \sqrt{90} \) as \( \sqrt{9} \) and \( \sqrt{10} \) to help us simplify
\[
\frac{3\sqrt{10}}{15} \quad \text{*Simplify } \sqrt{9} \\
= \frac{\sqrt{10}}{5} \quad \text{*Simplify the whole numbers (the 3 and 15) by dividing both the numerator and denominator by 3}
\]

Because we cannot simplify any further, \( \frac{\sqrt{10}}{5} \) is our final answer.

Simplify the following expression: \( \frac{x^2}{\sqrt{2x}} \)

\[
\frac{x^2}{\sqrt{2x}} = \frac{x^2 \cdot \sqrt{2x}}{\sqrt{2x} \cdot \sqrt{2x}} \quad \text{* Multiply both numerator and denominator by } \sqrt{2x} \\
= \frac{x^2 \sqrt{2x}}{2x} \quad \text{* Now we check to see whether anything can be simplified. In this problem, we can take an } x \text{ out of the numerator and the denominator} \\
= \frac{x \sqrt{2x}}{2}
\]

Because we cannot simplify any further, \( \frac{x \sqrt{2x}}{2} \) is our final answer.

Cube roots:
- Multiply the numerator and denominator by a factor that will create a perfect cube in the denominator. Then simplify.

Examples

Simplify the expression: \( \frac{4}{\sqrt[3]{6}} \)

\[
\frac{4}{\sqrt[3]{6}} = \frac{4 \cdot \sqrt[3]{6^2}}{\sqrt[3]{6} \cdot \sqrt[3]{6^2}} \quad \text{*Because we have one cube root of 6 in the denominator, we multiply by 2 more cube root of 6’s (or } \sqrt[3]{6^2} \text{) to create a perfect cube in the denominator.} 
\]
Under the cube root, we now have $6^2 = 36$, and we look to see if there is more we can simplify.

Divide both the numerator and denominator by 2 so that we can simplify as much as possible.

Because we cannot simplify any further, $\frac{2\sqrt[3]{36}}{3}$ is our final answer.

Simplify the expression: $\frac{7\sqrt[3]{xy}}{x^2}$

Because we have an $x^2$ in our denominator, we need one more $x$ to make it a perfect cube. Thusly, we multiply by $\sqrt[3]{x}$ on both the numerator and the denominator.

We look to see if we can simplify further. In this case, we cannot.

Because we cannot simplify any further, $\frac{7\sqrt[3]{x}}{x}$ is our final answer.

Simplify the expression: $\frac{\sqrt[3]{21y}}{\sqrt[3]{3xy^2}}$

In our denominator, we have one 3, one x, and two y’s. Thusly, we need two 3’s (or $3^2$, which is 9), two x’s (or $x^2$), and one y so that they all become perfect cubes. This means we will multiply the numerator and denominator each by $\sqrt[3]{9x^2y}$

Now we look to see if we are able to simplify any further. We want to specifically check if $\sqrt[3]{189}$ can be broken down further by taking out a perfect cube, which in this case it can be, using $\sqrt[3]{27}$ and $\sqrt[3]{7}$.
\[
\sqrt{\frac{3^2 \cdot 27}{3 \cdot xy}} = \sqrt{\frac{3^3 \cdot 7x^2 y}{3 \cdot xy}} = \sqrt{\frac{3^2 \cdot 7x^2 y}{xy}}
\]

*Split up \(\sqrt{189}\) as \(\sqrt{27}\) and \(\sqrt{7}\)

*Simplify \(\sqrt{27}\)

*Cancel out the 3’s in both the numerator and denominator

Because we cannot simplify any further, \(\frac{\sqrt{7} \cdot 3^2 \cdot x^2 y}{xy}\) is our final answer.

Denominators with two terms

- Multiply the numerator and denominator by the conjugate of the denominator. Make sure to distribute or FOIL the numerator and denominator. Then simplify.

Examples

Simplify the expression: \(\frac{5}{3 - \sqrt{2}}\)

\[
\frac{5}{3 - \sqrt{2}} = \frac{5}{3 - \sqrt{2}} \cdot \frac{3 + \sqrt{2}}{3 + \sqrt{2}} = \frac{15 + 5\sqrt{2}}{9 + 3\sqrt{2} - 3\sqrt{2} - 2} = \frac{15 + 5\sqrt{2}}{7}
\]

*Multiply by the conjugate

*FOIL the denominator, and distribute the numerator

*Simplify the denominator

Because we cannot simplify any further, \(\frac{15 + 5\sqrt{2}}{7}\) is our final answer.

Simplify the expression: \(\frac{6 + \sqrt{10}}{5 + \sqrt{6}}\)

\[
\frac{6 + \sqrt{10}}{5 + \sqrt{6}} = \frac{6 + \sqrt{10}}{5 + \sqrt{6}} \cdot \frac{5 - \sqrt{6}}{5 - \sqrt{6}} = \frac{6 + \sqrt{10} \cdot (5 - \sqrt{6})}{5^2 - (\sqrt{6})^2} = \frac{6 + \sqrt{10} \cdot 5 - \sqrt{10} \cdot \sqrt{6}}{25 - 6} = \frac{6 + 5\sqrt{10} - \sqrt{60}}{19}
\]

*Multiply by the conjugate

*Simplify the numerator and denominator...
\[
\frac{30 - 6\sqrt{6} + 5\sqrt{10} - \sqrt{60}}{25 - 5\sqrt{6} + 5\sqrt{6} - 6} = \frac{30 - 6\sqrt{6} + 5\sqrt{10} - 2\sqrt{15}}{19} \]

FOIL in both the numerator and denominator

* Simplify in both the numerator and the denominator. \(\sqrt{60} = 2\sqrt{15}\) in the numerator, and the \(\sqrt{6}\) terms cancel out in the denominator.

Because we cannot simplify any further, \(\frac{30 - 6\sqrt{6} + 5\sqrt{10} - 2\sqrt{15}}{19}\) is our final answer.

Simplify the expression:

\[
\frac{1 + \sqrt{3}}{1 - \sqrt{3}}
\]

\[
\frac{1 + \sqrt{3}}{1 - \sqrt{3}} = \frac{1 + \sqrt{3}}{1 - \sqrt{3}} \cdot \frac{1 + \sqrt{3}}{1 + \sqrt{3}} = \frac{1 + 2\sqrt{3} + 3}{1 + 3 - 2} = \frac{4 + 2\sqrt{3}}{-2}
\]

* Multiply by the conjugate

* FOIL in both the numerator and the denominator

* Combine like terms

* Simplify further by dividing out a common factor. In this case, we divide by -2, so that we no longer have a denominator (or in other words, our denominator is 1)

Because we cannot simplify any further, \(-2 - \sqrt{3}\) is our final answer.