Stats Review
Chapter 9
Note:

This review is meant to highlight basic concepts from the course. It does not cover all concepts presented by your instructor. Refer back to your notes, unit objectives, handouts, etc. to further prepare for your exam.

The questions are displayed on one slide followed by the answers are displayed in **red** on the next.

This review is available in alternate formats upon request.
Confidence Interval for Population Proportion

In a survey of 10 musicians, 2 were found to be left-handed. Is it practical to construct the 90% confidence interval for the population proportion, \( p \)?
Confidence Interval for Population Proportion

In a survey of 10 musicians, 2 were found to be left-handed. Is it practical to construct the 90% confidence interval for the population proportion, $p$?

- **Condition 1: $n(.05) \leq N$**
  - The sample size (10) is less than 5% of the population (millions of musicians), so the condition is met.
- **Condition 2: $np(1-p) \geq 10$**
  - $\hat{p} = \frac{2}{10} = .2$
  - $n\hat{p}(1 - \hat{p}) = 10(.2)(1 - .2) = 1.6$. This is less than 10 so this condition is not met.

**It would not be practical to construct the confidence interval.**
A poll conducted found that 944 of 1748 adults do not believe that people with tattoos are more rebellious. If appropriate construct a 90% confidence interval.
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Is it appropriate? Yes, it satisfies both conditions.

1) point estimate
\[ \hat{p} = \frac{944}{1748} = .54 \]

2) Find \( z_{\alpha/2} \). First take \( \frac{1-\cdot.90}{2} = .05 \)

   The corresponding \( z \) is -1.645. We will ignore the negative and just use 1.645.

3) Find the Margin of Error
\[ E = z_{\alpha/2} \sqrt{\frac{\hat{p}(1-\hat{p})}{n}} = 1.645 \sqrt{\frac{.54(1-.54)}{1748}} = .0196 \]

4) Take the point estimate and add/subtract the margin of error
\[ .54-.0196=.5204, \ .54+.0196=.5596 \]

The confidence interval is (.5204,.5596)
A researcher wants to estimate the proportion of Americans that have sleep deprivation. How large a sample is needed in order to be 95% confident and within 3% if

a) the researcher used a previous estimate of 60%?

b) the researcher doesn’t use a previous estimate?
A researcher wants to estimate the proportion of Americans that have sleep deprivation. How large a sample is needed in order to be 95% confident and within 3% if

a) the researcher used a previous estimate of 60%?

\[
N = \hat{p}(1 - \hat{p}) \left( \frac{Z_{\alpha/2}}{E} \right)^2 = .60(1-.60) \left( \frac{1.96}{.03} \right)^2 \approx 1024.4 \approx 1025
\]

b) the researcher doesn’t use a previous estimate?

\[
N = .25 \left( \frac{Z_{\alpha/2}}{E} \right)^2 = .25 \left( \frac{1.96}{.03} \right)^2 \approx 1067.1 \approx 1068
\]

*Remember to always round up when finding sample sizes.*
Find the t-value (using the table)

a) Right tail=.1, n=6

b) Left tail=.05, n=16

c) 90% confidence, n=21

d) 95% Confidence , n=83

e) 99% Confidence, n=1200
Find the t-value (using the table)

a) Right tail=.1, n=6
   Degrees of Freedom (df)=n-1=5, \( t = 1.476 \)

b) Left tail=.05 n=16
   \( df=15, t=-1.753 \) (t is negative for left tail)

c) 90% confidence, n=21
   \( df=20, \frac{1-.90}{2} = .05, t = 1.725 \)

d) 95% Confidence , n=83
   \( df= 82 \) (this is not in the table so choose the closest one: 80),
   \( \frac{1-.95}{2} = .025, t = 1.990 \)

e) 99% Confidence, n=1200
   \( df=1199, \) since this is more than 1000 we use the z-value
   \( \frac{1-.99}{2} = .005, t \ (or \ z) = 2.576 \)
Estimating a population mean

In a sample of 81 SARS patients, the mean incubation period was 4.6 days with a standard deviation of 15.9 days. Construct a 95% confident interval and interpret.
Estimating a population mean

In a sample of 81 SARS patients, the mean incubation period was 4.6 days with a standard deviation of 15.9 days. Construct a 95% confident interval and interpret.

• We cannot use $z$ because we do not have the population standard deviation.
• Find $t_{\alpha/2}$:

$$\frac{1-.95}{2} = .025, \text{ degrees of freedom } = n-1=81-80. \text{ The corresponding t-value is } 1.99$$

• Find the Margin of Error

$$E = t_{\alpha/2} \frac{s}{\sqrt{n}} = 1.99 \frac{15.9}{\sqrt{81}} = 3.516$$

• Add/subtract the margin of error from the sample mean

$$4.6-3.516 = 1.084, \ 4.6+3.516 = 8.116$$

We are 95% confident that the mean incubation period for SARS patients is between 1.084 and 8.116.
How large must a sample be in order to be 95% confident within 2 points given a sample standard deviation of 13.67?
Sample Size for Estimating Population Mean

How large must a sample be in order to be 95% confident within 2 points given a sample standard deviation of 13.67?

\[ n = \left( \frac{z \cdot s}{E} \right)^2 = \left( \frac{1.96 \cdot 13.67}{2} \right)^2 \approx 179.5 \approx 180 \]
Width of the Confidence Interval
What Happens to the width of the Confidence Interval

a) As the sample size increases

b) As the level of confidence increase
Width of the Confidence Interval

What Happens to the width of the Confidence Interval

a) As the sample size increases
   The width decreases

b) As the level of confidence increase
   The width Increases

To find out which margin of error is smaller (smaller width) with different sample sizes or level of confidence when everything else is the same, calculate

$$\frac{z_{\alpha/2}}{\sqrt{n}} \quad \text{ (for proportion)} \quad \text{or} \quad \frac{t_{\alpha/2}}{\sqrt{n}} \quad \text{ (for mean)}$$

for both cases to see which is smaller.
A 90% confidence interval for the hours that college students sleep during the weekday is (6.8, 10.8). Which interpretation(s) are correct?

a) 90% of college students sleep between 6.8 and 10.8 hours

b) We’re 90% confident that the mean number of hours of sleep that college students get any day of the week is between 6.8 and 10.8 hours

c) There is a 90% probability that the mean hours of sleep that college students get during a weekday is between 6.8 and 10.8 hours.

d) We’re 90% confident that the mean number of hours of sleep that college students during a weekday is between 6.8 and 10.8 hours.
A 90% confidence interval for the hours that college students sleep during the weekday is (6.8, 10.8). Which interpretation(s) are correct?

a) 90% of college students sleep between 6.8 and 10.8 hours
   Flawed: makes an implication about individuals rather than the mean

b) We’re 90% confident that the mean number of hours of sleep that college students get any day of the week is between 6.8 and 10.8 hours
   Flawed: should be about the weekday, not any day of the week

c) There is a 90% probability that the mean hours of sleep that college students get during a weekday is between 6.8 and 10.8 hours.
   Flawed: implies the population mean varies rather than the interval

d) We’re 90% confident that the mean number of hours of sleep that college students during a weekday is between 6.8 and 10.8 hours.
   Correct!
Find $\chi^2_{1-\alpha/2}$ and $\chi^2_{\alpha/2}$

90% confidence, n=20

95% confidence, n=25
Find $\chi^2_{1-\alpha/2}$ and $\chi^2_{\alpha/2}$

90% confidence, n=20

\[ \alpha = 1 - .90 = .1; \text{ df} = n-1 = 19 \]

- $\chi^2_{1-\alpha/2} = \chi^2_{1-.05/2} = \chi^2_{.95} = 10.117$
- $\chi^2_{\alpha/2} = \chi^2_{.5/2} = \chi^2_{.05} = 30.144$

95% confidence, n=25

\[ \alpha = 1 - .95 = .05; \text{ df} = n-1 = 24 \]

- $\chi^2_{1-\alpha/2} = \chi^2_{1-.025/2} = \chi^2_{.975} = 12.401$
- $\chi^2_{\alpha/2} = \chi^2_{.05/2} = \chi^2_{.025} = 39.364$

- If the degrees of freedom is not on the table, use the closest degrees of freedom
- If the degrees of freedom is directly between 2 values, find the mean of the values. Ex: For 65, take the $\chi^2$ values for both 60 and 70 and average their $\chi^2$ values.
A student randomly selects 10 paperback books at a store. The mean price is $8.75 with a standard deviation of $1.50. Construct a 95% confidence interval for the population standard deviation, $\sigma$. Interpret the confidence interval. Assume the data are normally distributed.
Estimating a Population Standard Deviation

A student randomly selects 10 paperback books at a store. The mean price is $8.75 with a standard deviation of $1.50. Construct a 95% confidence interval for the population standard deviation, \( \sigma \). Interpret the confidence interval. Assume the data are normally distributed.

1. Find \( \chi^2_{1-\alpha/2} \) and \( \chi^2_{\alpha/2} \) for 95% confidence level and df=9
   
   \[
   \chi^2_{1-.05/2} = 2.7 \\
   \chi^2_{.05/2} = 19.023
   \]

2. Find the lower and upper bounds of \( \sigma^2 \)
   
   Lower: \( \frac{(n-1)s^2}{\chi^2_{\alpha/2}} = \frac{(10-1)1.5^2}{19.023} = 1.065 \)
   
   Upper: \( \frac{(n-1)s^2}{\chi^2_{1-\alpha/2}} = \frac{(10-1)1.5^2}{2.7} = 7.5 \)

3. Since we want the confidence interval for \( \sigma \) and we have the lower/upper bounds of \( \sigma^2 \), we need to take the square roots of the lower and upper bound.
   
   Lower: \( \sqrt{1.065} = 1.03 \)
   
   Upper: \( \sqrt{7.5} = 2.74 \)

The confidence interval is \((1.03, 2.74)\)

We’re 95% confident that the population standard deviation of price of paperback books at this store is between $1.03 and $2.74 hours.